Low temperature properties of a quantum particle coupled to dissipative environments

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We study the dynamics of a quantum particle coupled to dissipative (ohmic) environments, such as an electron liquid. For certain choices of couplings, the properties of the particle can be described in terms of an effective mass. A particular case is the three dimensional dirty electron liquid. In other environments, like the one described by the Caldeira-Leggett model, the effective mass diverges at low temperatures, and quantum effects are strongly suppressed. For interactions within this class, arbitrarily weak potentials lead to localized solutions. Particles bound to external potentials, or moving in closed orbits, can show a first order transition, between strongly and weakly localized regimes.

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I. INTRODUCTION

The suppression of quantum effects by dissipative environments is a subject of current debate. It is known that in certain cases, like a quantum particle tunneling in a two level system[1] or in a periodic potential[2], whose coupling to the environment can be described by the Caldeira-Leggett model[3], quantum effects can be almost completely suppressed at low temperatures. On the other hand, perturbative calculations for the dephasing rate of electrons in a dirty Fermi liquid suggest that dephasing becomes irrelevant at low temperatures[4, 5]. Recent experiments[6], see also[7], have increased the interest in this topic, and a variety of theoretical analyses, with different conclusions, have followed[8, 9, 10, 11, 12, 13, 14].

The simplest situation where the effect of the dissipation can be studied is that of a single particle interacting with an external environment. A particular case is the Caldeira-Leggett model, where a specific choice of environment and coupling is made. This model has been extensively studied, and it is known that the diffusion of the particle at long times cannot be expressed in terms of a finite effective mass. In the following, we analyze related models which describe environments such as a dirty electron liquid. The results can be useful in understanding the quantum properties of external particles, such as protons or muons, at surfaces or inside metals[15, 16, 17]. In addition, the models studied here provide the simplest examples where the effects of the environment on quantum effects can be studied in the strong coupling, non perturbative, regime.

The next section describes the general features of the model. The method of calculation is discussed next. Section IV presents the results for a particle moving in free space. Section V includes extensions for particles localized around external potentials, or moving in bound orbits. A comparison with perturbation theory is presented in Section VI. The main conclusions of the present work are discussed in section VI.

II. THE MODEL

A convenient scheme to treat the problem of a particle interacting with an external environment is to integrate out the environment, and to analyze the resulting dynamics of the particle using the path integral formulation of quantum mechanics. The effect of the environment is to induce a retarded interaction among different positions $\vec{\mathbf{X}}(\tau), \vec{\mathbf{X}}(\tau')$ along a given path. The coupling to a given environment is defined as ohmic if this interaction decays as $(\tau-\tau')^{-2}$ at long times. In this case, the dynamics of the particle in the classical limit, $\hbar^2/M \to 0$, where M is the mass of the particle, can be described in terms of a macroscopic friction coefficient, η, which is finite as the velocity of the particle approaches zero (see below).

The effective interaction mediated by the environment can be expressed in terms of the response function of the environment, assuming that the coupling of the particle to each individual excitation is weak[3], or, alternatively, that the environment is weakly perturbed. In the following, we assume that the environment is an electron liquid, and that the particle is coupled, by a local potential, to the electronic charge fluctuations[18]:

$$\mathcal{H}_{int} = \int d^3\vec{\mathbf{x}} V\left(\vec{\mathbf{x}} - \vec{\mathbf{X}}\right)\rho(\vec{\mathbf{x}}) \quad (1)$$

where $\vec{\mathbf{X}}$ is the coordinate of the particle, $\rho(\vec{\mathbf{x}})$ describes the charge fluctuations of the electrons, and the V is the coupling potential. The induced retarded putential can be written as:

$$\mathcal{V}\left[\vec{\mathbf{X}}(\tau) - \vec{\mathbf{X}}(\tau')\right] = \int d\vec{\mathbf{q}} \int d\omega e^{i\vec{\mathbf{q}}\left[\vec{\mathbf{X}}(\tau)-\vec{\mathbf{X}}(\tau')\right]} e^{i\omega(\tau-\tau')} V^2(\vec{\mathbf{q}}) \mathrm{Im}\chi(\vec{\mathbf{q}},\omega) \quad (2)$$

where $\mathrm{Im}\chi(\vec{\mathbf{q}},\omega)$ is the Fourier transform of:

$$\mathrm{Im}\chi(\vec{\mathbf{x}}-\vec{\mathbf{x}}',\tau-\tau') = \langle\rho(\vec{\mathbf{x}},\tau)\rho(\vec{\mathbf{x}}',\tau')\rangle \quad (3)$$

For an electron liquid we have: $\lim_{\omega\to 0}\mathrm{Im}\chi(\vec{\mathbf{q}},\omega)\propto|\omega|$, and that fixes the long time behavior of the retarded interaction, eq.(2), which decays as $(\tau-\tau')^{-2}$. Finally, we can write for the effective action of the particle[18, 19]:

$$S = \int d\tau \frac{M}{2}\left(\frac{\partial\vec{\mathbf{X}}}{\partial\tau}\right)^2 + \int d\tau d\tau' \frac{\mathcal{F}\left[\left|\vec{\mathbf{X}}(\tau)-\vec{\mathbf{X}}(\tau')\right|^2\right]}{|\tau-\tau'|^2} \quad (4)$$

The function $\mathcal{F}$ is determined by the coupling between the particle and the electronic charge fluctuations.

The Caldeira-Leggett model can also be written in the form given in eq.(4), where the function $\mathcal{F}$ is $\mathcal{F}\propto\eta|\vec{\mathbf{X}}(\tau)-\vec{\mathbf{X}}'(\tau')|^2$, and η is the macroscopic friction coefficient. Other generalizations of the Caldeira-Leggett model, used in relation to Coulomb blockade, include higher order terms in the collective coordinate[20, 21], associated with higher order tunneling processes. These terms typically involve many different times as well. The derivation of eq.(4), using second order perturbation theory, leads to non linear terms which couple the coordinates at two different times only.

In general, we can write the function $\mathcal{F}$ as:

$$\mathcal{F}(u) = \alpha f\left(\frac{u}{l^2}\right) \quad (5)$$

where α is a dimensionless constant and l is a length scale typical of the fluctuations in the environment. We assume, without loss of generality, that $f(0)=0$ and $f'(0)=1$. At high temperatures, where $l_T=\sqrt{\hbar^2/(MT)}\ll l$, the motion of the particle is determined by $\lim_{u\to 0}\mathcal{F}(u)\approx\alpha/l^2 f'(0)$. In this limit, eq.(4) is equivalent to the Caldeira-Leggett model with a friction coefficient $\eta=\alpha/l^2$.

In the following, we will assume that the response function of the environment is that of a dirty (diffusive) electron liquid, and the external particle couples to the charge fluctuations via a screened Coulomb potential. Then:

$$\begin{aligned}\chi_0(\vec{\mathbf{q}},\omega) &\approx \nu\frac{\mathcal{D}|\vec{\mathbf{q}}|^2}{i\omega+\mathcal{D}|\vec{\mathbf{q}}|^2}\\ \mathcal{F}(\vec{\mathbf{q}}) &\approx \frac{1}{\nu\mathcal{D}|\vec{\mathbf{q}}|^2}\end{aligned} \quad (6)$$

where $\mathcal{F}(\vec{\mathbf{q}})$ is the Fourier transform of $\mathcal{F}\left[\vec{\mathbf{X}}(\tau)-\vec{\mathbf{X}}'(\tau')\right]$. In the calculation of $\mathcal{F}$ we have included the full selfconsistent screened Coulomb potential. The expressions in eqs.(6) determine the the function $\mathcal{F}$ in eq.(4) for distances greater than the mean free path, l. At shorter distances, we will choose a regularization consistent with the expected asymptotic regime. Finally, we will compare the results with two other choices of the retarded interaction: i) the one appropiate for the Caldeira-Leggett model, and ii) a retarded interaction which decays exponentially beyond a certain length, l[22]. The retarded actions to be considered, expressed in terms of the function f given in eq.(5), are:

$$\begin{aligned}
f(u) &= u && \text{Caldeira}-\text{Leggett model}\\
f(u) &= 1-e^{-u} && \text{short range kernel}\\
f(u) &= 2\sqrt{1+u}-2 && \text{1D dirty electron gas}\\
f(u) &= \log(1+u) && \text{2D dirty electron gas}\\
f(u) &= 2-\tfrac{2}{\sqrt{1+u}} && \text{3D dirty electron gas}
\end{aligned} \quad (7)$$

Combining eq.(5) and eqs.(7), the macroscopic friction coefficient is, in all cases, $\eta=\alpha/l^2$. For the dirty electron gas, $\alpha\approx l^{2-D}/(\nu\mathcal{D})$. This value can change if the particle is coupled to different electronic reservoirs. For a clean electron gas, α can be written in terms of the phaseshifts induced by the presence of the particle on the electrons at the Fermi level[28]. In the following, we will treat α as a variable which can take arbitrary values.

III. THE CALCULATION

A. Perturbative expansion.

The action described in eq.(4) can be solved exactly when the retarded interaction is given by the Caldeira-Leggett model. In this case, the action depends quadratically on the spatial coordinate, $\vec{\mathbf{X}}(\tau)$. In the following, we will assume that τ is the imaginary time.

In general, one can write the expansion:

$$S = \int d\tau \frac{M}{2} \left(\frac{\partial \vec{\mathbf{X}}}{\partial \tau} \right)^2 + \sum_n \alpha_n \int d\tau d\tau' \frac{\left| \vec{\mathbf{X}}(\tau) - \vec{\mathbf{X}}(\tau') \right|^{2n}}{|\tau - \tau'|^2} \tag{8}$$

where:

$$\alpha_n = \alpha \frac{1}{n! l^{2n}} \frac{\partial^n f}{\partial u^n} \tag{9}$$

The case where only the $n = 1$ term in the sum in eq.(8) is non zero corresponds to the Caldeira-Leggett model. One can write a perturbative expansion for the corrections due to the $n > 1$ terms. Typical diagrams are given in Fig.[1]. It is easy to show that all $\alpha_n, n > 1$ acquire logarithmic corrections. In Renormalization Group terms, they are all marginal. In this respect, the model differs from standard models in statistical mechanics, where usually only quartic terms need to be considered. If we only consider the quartic term, α_2, the logarithmic divergence of the perturbative series lead to the scaling equations (see Fig.[1]):

$$\begin{aligned} \frac{\partial \alpha_1}{\partial l} &= \frac{2\alpha_2}{\alpha_1} \\ \frac{\partial \alpha_2}{\partial l} &= -\frac{4\alpha_2^2}{\alpha_1^2} \end{aligned} \tag{10}$$

where $l = \log(\Lambda_0/\Lambda)$, Λ_0 is the initial high energy (short time) cutoff needed to regularize eq.(4), and Λ is the effective cutoff. A scaling approach following eq.(10) is, however, impractical, because one should consider an infinite set of coupled equations, including all the couplings.

B. Large N approximation.

The non linear terms in the action in eq.(8) are greatly simplified when the number of components N of the vector $\vec{\mathbf{X}}$ is large. In order to have a consistent theory, we must rescale the argument u of the function $f(u)$ in eq.(5) so that $f(u) = N f(\bar{u}/N)$, where $\bar{u}$ is proportional to $|\vec{\mathbf{X}}(\tau) - \vec{\mathbf{X}}'(\tau')|^2$. In terms of the diagrammatic expansion sketched in Fig.[1], we need only to consider closed loop diagrams, like those in Fig.[2]. These diagrams can be summed using standard large-N techniques in statistical mechanics[23, 24, 25], leading to the equations:

$$\begin{aligned} \Sigma(\tau) &= \frac{\alpha}{\tau^2} f'[G(\tau)] \\ G(\omega) &= \frac{l^2}{\frac{M l^2 \omega^2}{2} + \Sigma(\omega)} \end{aligned} \tag{11}$$

where:

$$G(\tau) = \left\langle \left| \vec{\mathbf{X}}(\tau) - \vec{\mathbf{X}}(0) \right|^2 \right\rangle \tag{12}$$

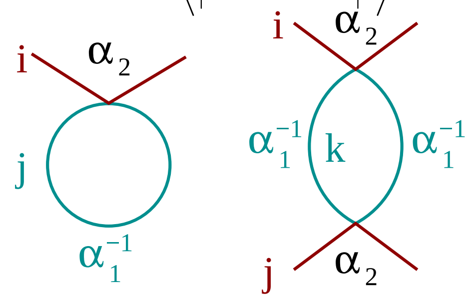

FIG. 1: Some simple diagrams which renormalize the values of α_1 and α_2 in eq.(8). The propagators are proportional to α_1^{-1}. Indices denote components of the vector $\vec{\mathbf{X}}$.

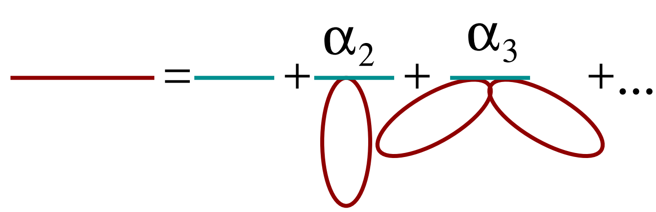

FIG. 2: Sketch of the selfconsistent solution of closed loop diagrams which leads to eqs.(11).

and, using eq.(11), we can write:

$$G(\tau) = l^2 \bar{G} \left[\alpha, \frac{\hbar^2}{2Ml^2} \tau \right] \tag{13}$$

Two exactly solvable cases are:

$$G(\tau) \propto \begin{cases} \frac{\tau}{M} & \text{free particle of mass } M \\ \frac{1}{\eta} \log\left[\frac{\eta\tau}{M}\right] & \text{Caldeira} - \text{Leggett model} \end{cases} \tag{14}$$

C. Variational ansatz.

The large-N expansion described in the previous subsection can be alternatively formulated as a variational approximation to the action in eq.(4). We use the ansatz:

$$S_0 = \int d\tau \frac{M}{2}\left(\frac{\partial \vec{\mathbf{X}}}{\partial \tau}\right)^2 + \int d\tau d\tau' \left|\vec{\mathbf{X}}(\tau) - \vec{\mathbf{X}}(\tau')\right|^2 \Sigma(\tau - \tau') \tag{15}$$

where $\Sigma(\tau)$ is a function to be determined from the minimization of $\langle S - S_0\rangle_0 + F_0$. The subscript 0 means averaging with respect to S_0, and F_0 is the free energy associated to S_0.

The function $\Sigma(\tau)$ satisfies the equation:

$$\Sigma(\tau) = \frac{1}{\tau^2}\frac{\partial}{\partial G_0(\tau)}\left\langle \mathcal{F}\left[\left|\vec{\mathbf{X}}(\tau) - \vec{\mathbf{X}}(0)\right|^2\right]\right\rangle_0 \tag{16}$$

and:

$$G_0(\tau) = \left\langle \left|\vec{\mathbf{X}}(\tau) - \vec{\mathbf{X}}(0)\right|^2\right\rangle_0 \tag{17}$$

Eqs. (16) and (17) are equivalent to eqs.(11). The advantage of the variational formulation is that it can be extended to finite values of N. The expectation values to be calculated are of the type:

$$\left\langle \mathcal{F}\left[\left|\vec{\mathbf{X}}(\tau) - \vec{\mathbf{X}}(0)\right|^2\right]\right\rangle_0 = C_N \int_0^\infty dr r^{N-1}\mathcal{F}(r^2)e^{-r^2/[2G_0(\tau)]} \tag{18}$$

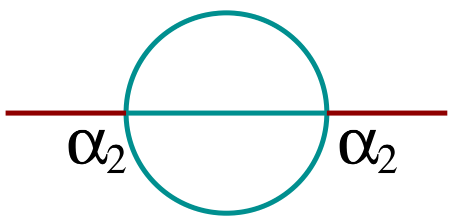

FIG. 3: Simplest diagram which gives a vertex correction to the variational scheme discussed in the text.

where:

$$\begin{aligned} C_N &= \sqrt{\pi}\sqrt{G_0(\tau-\tau')} \quad N=1 \\ C_N &= \sqrt{\pi}[G_0(\tau-\tau')]^{3/2} \quad N=3 \end{aligned} \tag{19}$$

D. Vertex corrections.

For the physically relevant class of $N = 2, 3$, the above approximations neglect vertex corrections, like those shown in Fig.[3]. In the next section, we will analyze them, and argue that they do not change qualitetively the solutions.

IV. RESULTS. FREE PARTICLE

A. Parameters of the model.

Eqs. (11) can be solved selfconsistently. The model has two dimensionless parameters, the value of α, defined in eqs.(5,7), and the ratio $\hbar^2/(Ml^2\Lambda_0)$, where Λ_0 is the cutoff in the spectrum of the environment, which we will assume to be finite. Note, however, that, for most physical quantities, the limit $\hbar^2/(Ml^2\Lambda_0) \to 0$ is well defined. The scaling given in eq.(13) allows us to set $M = 1$ and $l = 1$.

B. Asymptotic analysis.

We first analyze the selfenergy correction due to the environment, $\Sigma(\omega)$ in eq.(11). At very short times, $\tau \ll \tau_0\Lambda_0^{-1}$, the environment cannot influence the particle, and $\Sigma(\tau) = 0$. For times $\tau \gg \tau_0$ such that $G(\tau)$ is small, $\left\langle \left|\vec{\mathbf{X}}(\tau) - \vec{\mathbf{X}}(\tau')\right|^2\right\rangle \ll l^2$. Then:

$$\mathcal{F}\left(\left|\vec{\mathbf{X}}\right|^2\right) \sim \alpha\frac{\left|\vec{\mathbf{X}}\right|^2}{l^2} \tag{20}$$

In this regime, the behavior of the system is indistinguishable from that of an effective Caldeira-Leggett

model, and:

$$\Sigma(\omega) \approx \frac{\alpha}{l^2}|\omega|$$
$$G(\tau) \approx \frac{l^2}{\alpha}\log\left(\frac{\alpha\tau}{Ml^2}\right) \tag{21}$$

This approximation is valid provided that $G(\tau) \ll l^2$. This contraint sets a maximum time, $\tau_1 \sim (Ml^2 e^{\alpha})/\alpha$, beyond which the diffusion of the particle cannot be described by eqs.(21). For $\tau \gg \tau_1$, $\Sigma(\omega)$ acquires a contribution:

$$\Sigma(\omega) = \int_{\tau_0}^{\tau_1} \frac{\alpha}{l^2}\frac{1-e^{i\omega\tau}}{\tau^2} + \Sigma_{\tau\gg\tau_1}(\omega) \sim \frac{\alpha}{l^2}\omega^2\tau_1 + \Sigma_{\tau\gg\tau_1}(\omega) \tag{22}$$

where a a term arising from times longer than τ_1, $\Sigma_{\tau\gg\tau_1}(\omega)$, has to be added.

$\Sigma_{\tau\gg\tau_1}(\omega)$ is determined by $\lim_{u\to\infty} f(u)$ in eqs.(7). If $\Sigma_{\tau\gg\tau_1}(\omega)$ goes to zero faster than ω^2 as $\omega \to 0$, then $G(\tau) \propto \tau$. Hence, we can check if the asumption that $G(\tau) \propto \tau$ leads to a self consistent solution. We insert this ansatz into the expression of $\Sigma_{\tau\gg\tau_1}(\tau)$. At long times, we have:

$$\lim_{\tau\to\infty} \Sigma(\tau) \propto \frac{\alpha}{\tau^2}\lim_{\tau\to\infty} f'[G(\tau)] \tag{23}$$

where c is a constant. If $f'[G(\tau)]$ decays faster than $G(\tau)^{-1}$, then $\lim_{\omega\to 0}\Sigma_{\tau\gg\tau_1}(\omega)$ goes to zero faster than ω^2.

Thus, if $\lim_{u\to\infty} f'(u)$ decays faster than u^{-1}, we can write:

$$G(\tau) \sim \int_0^{\tau_1^{-1}} d\omega \frac{1-e^{i\omega\tau}}{M\omega^2 + \omega^2\tau_1/l^2} \sim l^2\frac{\tau}{\tau_1} \sim Me^{\alpha}\tau \tag{24}$$

This equation describes the propagation of a quantum particle with effective mass $M_{eff} \sim Me^{\alpha}$.

In terms of the action in eq.(4), the previous analysis allows us to describe the dynamics of the quantum particle in terms of a renormalized effective mass when the function which defines the coupling to the environment is such that:

$$\lim_{\left|\vec{\mathbf{X}}(\tau)-\vec{\mathbf{X}}(\tau')\right|^2\to\infty} \left\langle \mathcal{F}\left[\left|\vec{\mathbf{X}}(\tau)-\vec{\mathbf{X}}(\tau')\right|^2\right]\right\rangle \sim o\left[\left|\vec{\mathbf{X}}(\tau)-\vec{\mathbf{X}}(\tau')\right|^{-2}\right] \tag{25}$$

This is the case when the environment is a three dimensional dirty electron liquid, or for a short range kernel, see eqs.(7). It is interesting to note that, for finite dimensions N, eq.(18) implies that $\left\langle \mathcal{F}\left[\left|\vec{\mathbf{X}}(\tau)-\vec{\mathbf{X}}(\tau')\right|^2\right]\right\rangle$ never decays faster than $\left|\vec{\mathbf{X}}(\tau)-\vec{\mathbf{X}}(\tau')\right|^{-N}$.

C. Vertex corrections.

The scheme used here neglects vertex corrections. Diagrams such as the one shown in Fig.[3] give a contribution, $\Sigma_{vertex}(\tau)$ to $\Sigma(\tau)$, and can change the above results if they decay at long times more slowly than τ^{-3}. The diagram in Fig.[3] goes as:

$$\Sigma_{vertex}(\tau) \sim \left\{f''\left[G(\tau)\right]\right\}^2 G^3(\tau) \tag{26}$$

If $G(\tau) \sim \tau$ and f is such that eq.(25) is satisfied, then $\Sigma_{vertex}(\tau)$ decays faster than τ^{-3} at long times, and the correction from the diagram in Fig.[3] does not change the results described above. It is easy to show that the same is also valid for more complicated vertex diagrams. This analysis leads us to conjecture that, when the dynamics of the particle can be described in terms of an effective mass, vertex corrections are irrelevant.

D. Simple actions.

The previous analysis allows us to study the simple case where $\mathcal{F}(\vec{\mathbf{X}}) = \alpha_n|\vec{\mathbf{X}}|^{2n}$. The case $n = 1$ corresponds to the Caldeira-Leggett model. For $n > 1$, we can make the ansatz $\lim_{\tau\to\infty} G(\tau) \propto a_n \log^{\gamma_n}(\tau)$. Then,

from eqs.(11):

$$\Sigma(\tau) \propto \frac{\alpha_n}{\tau^2} \left[a_n \log^{\gamma_n}(\tau) \right]^{n-1} \tag{27}$$

and:

$$G(\omega) \propto \frac{1}{\alpha_n |\omega| \left[a_n \log^{\gamma_n}(\omega) \right]^{n-1}} \tag{28}$$

so that:

$$\begin{aligned} \frac{1}{\alpha_n a_n^{n-1}} &= a_n \\ 1 + (1-n)\gamma_n &= \gamma_n \end{aligned} \tag{29}$$

and, finally:

$$\lim_{\tau \to \infty} G(\tau) \propto \left[\frac{1}{\alpha_n} \log(\tau) \right]^{1/n} \tag{30}$$

This correlation function implies that the particle is never localized, although it can diffuse more slowly than for the Caldeira-Leggett model. The effective mass diverges at low temperatures. The case $n = 1/2$ corresponds to the one dimensional dirty electron gas, described in eq.(7).

E. Numerical results.

We have solved iteratively eqs.(11) for different choices of the environment. The resulting correlation function $G(\tau)$ is shown in Figure[4] for the cases:

$$\begin{array}{lcl} f(x) = & \frac{1}{\sqrt{1+x}} & \text{(3D dirty electron gas)} \\ f(x) = & \log(1+x) & \text{(2D dirty electron gas)} \\ f(x) = & x & \text{(Caldeira – Leggett model)} \end{array} \tag{31}$$

The dimensionless parameters in the three cases are $\alpha = 2$ and $\hbar^2/(Ml^2\Lambda_0) = 0.05$. The units are such that $M = 1$ and $l = 1$. The macroscopic friction coefficient, $\eta = \alpha/l^2$ is the same in the three cases. It is clear that the spatial dependence of the retarded interaction induces significant differences in the long time dynamics of the particle, although look the same at sufficiently high temperatures, where the mass, and the macroscopic friction coefficient are the only relevant parameters.

For the Caldeira-Leggett model, $G(\tau)$ increases logarithmically at long times. The effects of the three dimensional dirty electron gas can be described in terms of an effective mass, which, for the parameters used here, is about two orders of magnitude larger than the bare mass. The two dimensional dirty electron gas is a marginal case, and the numerical results, like those shown in Fig.[6], suggest that $G(\tau) \propto \tau^\kappa$, with $0 < \kappa < 1$, and $\kappa \propto 1 - g^{-1}$, where g is the conductance.

The effective mass, when the environment is described by the dirty three dimensional liquid, as function of the strength coupling, α is shown in Fig.[5]. The numerical results support the exponential dependence on α discussed in the preceding subsection.

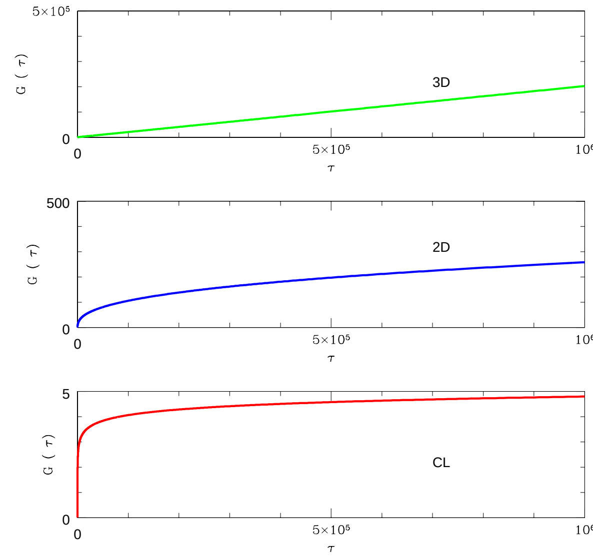

FIG. 4: Time dependence of the propagator $G(\tau)$, for $\hbar^2/(Ml^2) = 1$, $l = 1$ and $\alpha = 2$ for the three cases described in eq.(31).

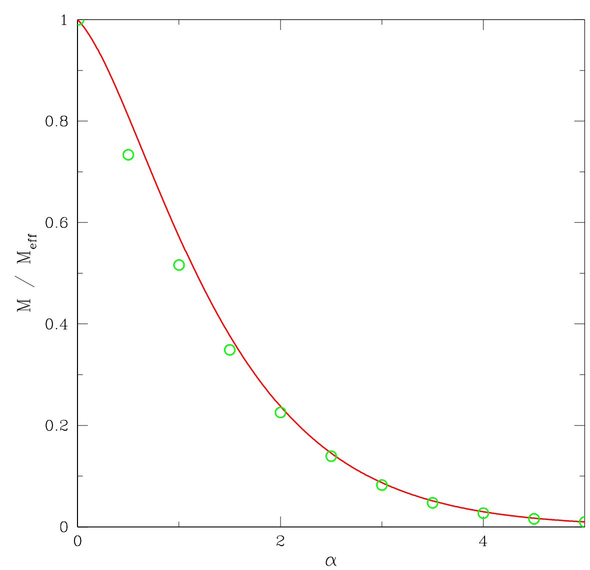

FIG. 5: Dots: Effective mass of a particle interacting with a dirty three dimensional electron liquid, as function of the strength of the interaction, α. The full line is a fit to a function of the type $M/M_{eff} = (c_1 + c_2\alpha)e^{-c_3\alpha}$.

V. RESULTS. LOCALIZED SOLUTIONS

A. Local attractive potential.

An external potential, V, acting on the particle can be described, using the path integral formulation, by adding

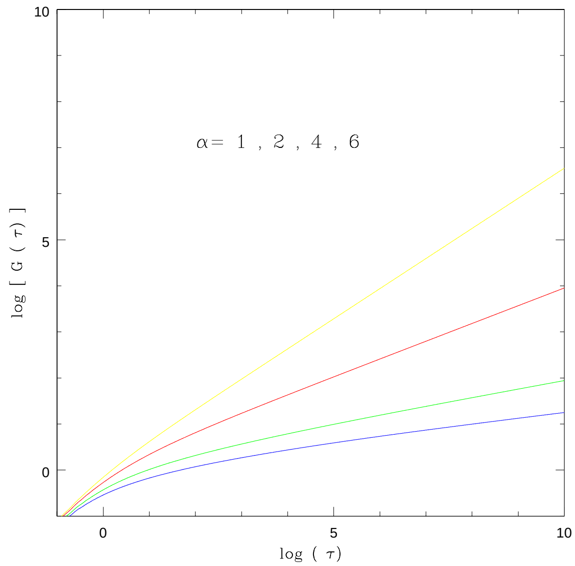

FIG. 6: Log-log plots of $G(\tau)$ vs. τ for an action which describes a two dimensional dirty electron liquid, eq.(7). From top to bottom: $\alpha = 1, 2, 4, 6$.

a term to the action in eq.(4):

$$S_{pot} = \int d\tau V \left[\vec{\mathbf{X}}(\tau)\right] \tag{32}$$

This term leads to new nonlinear effects. We can extend the large-N or variational scheme described earlier by using the approximate action:

$$S_{var} = \int d\tau \frac{\lambda}{2} \left|\vec{\mathbf{X}}(\tau)\right|^2 \tag{33}$$

where λ is a variational parameter. This ansatz, in the absence of dissipation, is equivalent to the use of a variational set of gaussian wavefunctions in order to probe the existence of bound states of the potential $V(\vec{\mathbf{X}})$. It is well known that this method can be used to study the existence of localized solutions for arbitrarily small potentials in one dimension, while a threshold is required for the existence of a bound state in three dimensions.

We now analyze the full action, given by eqs.(4) and (32) using the variational ansatz given by eqs.(15) and (33). We will focus on the possible existence of localized solutions, characterized by a finite value of λ, in the limit of a very weak potential $V(\vec{\mathbf{X}})$. Then, $\lambda l^2 \ll \Lambda_0, \hbar^2/(Ml^2)$, and the Fourier transform of $G(\tau)$ is:

$$G(\omega) = \frac{l^2}{\frac{Ml^2\omega^2}{2} + \Sigma_{\lambda=0}(\omega) + \lambda l^2} \tag{34}$$

where $\Sigma_{\lambda=0}(\omega)$ is the self energy calculated when $\lambda = 0$.

We now write the external potential as $V(\vec{\mathbf{X}}) = \bar{V}(u)$, where $u = |\vec{\mathbf{X}}|^2$. The self consistency equation for λ is:

$$\lambda = \frac{\partial}{\partial G_\infty} \left\langle \bar{V}\left(G_\infty\right)\right\rangle_{var} \tag{35}$$

The expectation value in the r.h.s. of this equation is to be calculated using the variational action, eqs. (15) and (33). This calculation is similar to that performed in eq.(18). We are interested in the limit when $G_\infty \gg l^2$. Then, for localized potentials:

$$\frac{\partial}{\partial G_\infty} \left\langle \bar{V}\left(G_\infty\right)\right\rangle_{var} \sim \frac{1}{G_\infty^{N/2+1}} \tag{36}$$

For the Caldeira-Leggett model, $G_\infty \propto \eta^{-1} \log[\eta^2/(M\lambda)]$. Combining eqs.(35) and (36), we find that there is a localized solution with a for any attractive potential $V(\vec{\mathbf{X}})$. The corresponding value of λ is:

$$\lambda \sim \left(\frac{\eta}{\hbar}\right)^{N/2+1} \int d^N \vec{\mathbf{X}} V(\vec{\mathbf{X}}) \tag{37}$$

The value $\Delta = \lambda/\eta$ can be interpreted as a gap in the spectrum of the excitations of the particle.

If the dynamics of the particle can be described in terms of an effective mass, $G_\infty \propto \sqrt{\hbar^2/(M_{eff}\lambda)}$. Then, eqs.(35) and (36), lead to:

$$\lambda \propto \lambda^{N/4+1/2} \int d^N \vec{\mathbf{X}} V(\vec{\mathbf{X}}) \tag{38}$$

This equation has solutions for arbitrarily weak potentials only if $N = 1$, that is, if the particle moves in one dimension, in agreement with standard quantum mechanics.

B. Closed orbits.

We can use the previous method to analyze the motion of the particle when it moves around closed circular loops. When the loop is threaded by a magnetic flux, this geometry can be used to analyze quantum interference effects[14, 26]. The trajectory of the particle can be described in terms of an angle, and $|\vec{\mathbf{X}}(\tau)| = R$, where R is the radius of the orbit. The action in eq.(4) describes a one dimensional non linear sigma model with long range interactions. The case when only quadratic terms are kept in the expansion in eq.(8) has been extensively discussed in the literature[27, 28]. The large-N extension of this model can be analyzed using standard techniques[29, 30]. The constraint can be incorporated by means of a Lagrange multiplier, $\lambda(\tau)$, whose fluctuations can be neglected in the large-N limit. This term in the action is given by eq.(33), and we can apply a similar scheme to that discussed in the previous subsection. The value of λR^2 gives the new energy scale which describes the dynamics of the particle around a closed loop. Alternatively, λR^2 can be interpreted as an inverse correlation

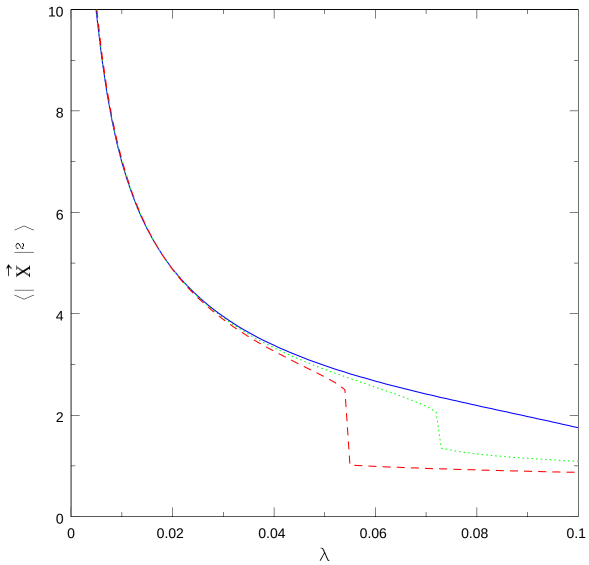

FIG. 7: First order transition obtained for $\Lambda_0/(\hbar^2/Ml^2) = 1$. The parameters are $M = 1$ and $l = 1$, and $f(u) = e^{-u}$ in eq.(5). Full curve: $\alpha = 6$. Dotted curve, $\alpha = 8$. Broken curve: $\alpha = 10$.

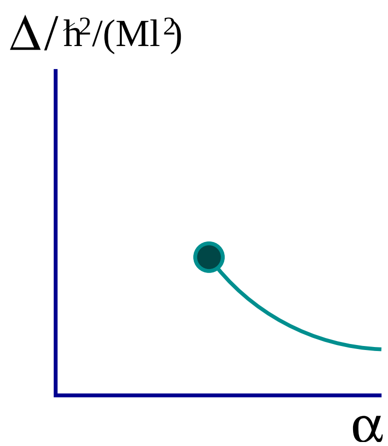

FIG. 8: Sketch of the transition discussed in the text.

time[29]. This approach describes well the phase transition of the model when the retarded interaction decays as $\tau^{-2+\epsilon}$[29, 30].

The dynamics of the particle, using the full action eq.(8), is determined by G_∞, for $R \gg l$. The value of λ is given by the solution of:

$$G_\infty \approx R^2 \tag{39}$$

For the Caldeira-Leggett model, this equation leads to $\lambda \propto e^{-(\eta R^2)/\hbar}$, as $R/l \to \infty$, in agreement with previous calculations[26, 31]. If the motion of the particle can be described in terms of an effective mass, $\lambda R^2 \sim \hbar^2/(M_{eff} R^2)$, also in agreement with earlier work[26].

C. First order phase transition.

The results discussed in the previous subsections are valid if the high energy cutoff of the bath, Λ_0, is much larger than the energy scale $\hbar^2/(Ml^2)$. When these scales become comparable, we find a first order phase transition, as function of λ or R between two regimes:

i) The particle moves away from the region $|\vec{\mathbf{X}}| < l$ at times shorter than Λ_0^{-1}, and, for longer times, it diffuses like a free particle.

ii) The particle is trapped inside the region $|\vec{\mathbf{X}}| < l$ for all times.

This transition is analogous to that described recently in the two dimensional non linear sigma model[32, 33, 34]. The transition takes place between two disordered phases, and it does not violate the Mermin-Wagner theorem, which can also be formulated for the model studied here. The phase diagram is sketched in Fig.[8]. There is a line of first order transitions, which ends at a critical point. The discontinuity at the phase transition increases as the minimum in the function $\mathcal{F}(\vec{\mathbf{X}})$ becomes deeper and more localized around $\vec{\mathbf{X}} = 0$.

The existence of a critical point, shown in Fig.[8], implies the existence of physical quantities with anomalous decay in the time domain. Following the results in[33], we conjecture that the energy-energy correlations will show power law correlations in time[35].

VI. COMPARISON WITH PERTURBATION THEORY

It is instructive to compare the results presented in the previous sections with a perturbative calculation, which is the scheme most widely used when studying dephasing[4, 5, 10]. We consider a particle in free space. The unperturbed states are plane waves, characterized by a momentum $\vec{\mathbf{k}}$, and an energy $\epsilon_{\vec{\mathbf{k}}} = |\vec{\mathbf{k}}|^2/M$. The inverse lifetime of this state, when the coupling to the environment is described by the action in eq.(4), is:

$$\Gamma_{\vec{\mathbf{k}}}(T) = \int d\omega \int d\vec{\mathbf{q}} \mathcal{F}(\vec{\mathbf{q}}) |\omega| \left[\left(1 + e^{-\omega/T}\right) \delta\left(\omega - \epsilon_{\vec{\mathbf{k}}} + \epsilon_{\vec{\mathbf{k}}+\vec{\mathbf{q}}}\right) + e^{-\omega/T} \delta\left(\omega + \epsilon_{\vec{\mathbf{k}}} - \epsilon_{\vec{\mathbf{k}}+\vec{\mathbf{q}}}\right)\right] \tag{40}$$

where $\mathcal{F}(\vec{\mathbf{q}})$ is the Fourier transform of $\mathcal{F}\left[\vec{\mathbf{X}}(\tau)-\vec{\mathbf{X}}(\tau')\right]$ in eq.(4). The two terms in the r.h.s. of eq.(40) describe the emission and absorption of quanta by the environment.

For the case of a dirty electron gas studied earlier, $\mathcal{F}(\vec{\mathbf{q}}) \approx 1/(|\vec{\mathbf{q}}|^2\nu\mathcal{D})$, where ν is the density of states, and $\mathcal{D}$ is the diffusion coefficient[26]. The integral in eq.(40) is convergent, except in one dimension. We can define selfconsistently a dephasing lifetime[4, 5, 10] where contributions from low energy transitions are suppressed. Then, when $\epsilon_{\vec{\mathbf{k}}} \ll T$, we find:

$$\lim_{T\to 0}\Gamma_{\vec{\mathbf{k}}}(T) \approx \frac{\hbar}{M\nu\mathcal{D}l_T^D} \propto T^{D/2} \tag{41}$$

where $l_T = \sqrt{\hbar^2/(MT)}$. There are logarithmic corrections for $D=1$, which arise from the divergence in the integral in eq.(40).

We can also calculate the real part of the self energy, which, at zero temperature, is:

$$\mathrm{Re}\Sigma\left(\vec{\mathbf{k}},\omega\right) = \int d\omega' \int d\vec{\mathbf{q}} \frac{\mathcal{F}(\vec{\mathbf{q}})|\omega'|}{\omega-\omega'+\epsilon_{\vec{\mathbf{k}}}-\epsilon_{\vec{\mathbf{k}}+\vec{\mathbf{q}}}} \tag{42}$$

Quantities associated to the derivatives of the real part of the self energy, like the effective mass renormalization and the quasiparticle residue:

$$\begin{aligned}
\frac{\delta M}{M^2} &= -\nabla^2_{\vec{\mathbf{k}}}\mathrm{Re}\Sigma\left(\vec{\mathbf{k}},\omega\right)\Big|_{\omega=\epsilon_{\vec{\mathbf{k}}}} \\
Z &= \left[1+\left.\frac{\partial\mathrm{Re}\Sigma\left(\vec{\mathbf{k}},\omega\right)}{\partial\omega}\right|_{\omega=\epsilon_{\vec{\mathbf{k}}}}\right]^{-1}
\end{aligned} \tag{43}$$

are divergent as $\vec{\mathbf{k}} \to 0$, for D=1,2.

Eq.(41) suggests that quantum effects are still important at zero temperature. This interpretation, however, is not consistent with the non perturbative results for $D=1,2$ (see subesections IVD and IVE). In both cases, we find that $G(\tau)$ increases more slowly than τ, so that the energy scale for the Aharonov-Bohm oscillations in a closed orbit acquire an anomalous R dependence (see IVB). We can infer an effective "dephasing time", from the scale at which $G(\tau)$ reaches its asymptotic unconventional behavior. In both cases, this scale is $\tau_\phi \propto (\hbar^2/Ml^2)^{-1}$, where l is the mean free path in the environment. The unperturbed case is recovered when $l\to\infty$.

The Caldeira-Leggett model leads to $\mathcal{F}(\vec{\mathbf{q}}) \approx \delta''(\vec{\mathbf{q}})$, and, for $\epsilon_{\vec{\mathbf{k}}} \ll T$, $\lim_{T\to 0}\Gamma_{\vec{\mathbf{k}}}(T) \approx \frac{\hbar\eta}{M}$. This result is consistent with the non perturbative, analysis of the model, which suggests that quantum effects are strongly suppressed, even at zero temperature.

VII. CONCLUSIONS

We have analyzed the low energy properties of a quantum particle interacting with dissipative environments, characterized by an ohmic response. By means of a large-N, or variational approximation, and numerical calculations, we have estimated time correlations which characterize the dynamics of the particle. Our method allows us to treat both the weak coupling limit, and the Caldeira-Leggett model, where the coupling to the environment strongly suppresses the quantum properties of the particle. The retarded interactions induced by the coupling to the environment are described by a function, $\mathcal{F}\left[|\vec{\mathbf{X}}(\tau)-\vec{\mathbf{X}}(\tau')|^2\right]$, which depends on the type of environment, eq.(4). For a particle propagating freely, we characterize the dynamics of the particle by the correlation function $G(\tau-\tau') = \left\langle\left[\vec{\mathbf{X}}(\tau)-\vec{\mathbf{X}}(\tau')\right]^2\right\rangle$.

We find that ohmic environments can be divided into two broad classes (even if they all give rise to the same macroscopic dissipative equations of motion):

i) If $\lim_{u\to\infty}\mathcal{F}'(u)$ decays faster than u^{-1}, the behavior of the particle can be described in terms of an effective mass, M_{eff}. At long times, $G(\tau) \propto M_{eff}\tau$. We have estimated the renormalization of the bare mass, which depends exponentially on the coupling. Environments with these features are the clean and dirty three dimensional electron liquids, and the short range kernel where $\mathcal{F}(u) \propto 1-e^{-u/l}$.

ii) If $\lim_{u\to\infty}\mathcal{F}'(u)$ decays more slowly that u^{-1}, the effective mass of the particle becomes infinite at zero temperature. At long times, $G(\tau) \propto \log^\gamma(\tau)$. This is the behavior of the Caldeira-Leggett model ($\gamma=1$), and the one dimensional dirty electron gas ($\gamma=1/2$).

The two dimensional dirty electron gas is an intermediate case. Our results are consistent with a power law dependence, $G(\tau)\propto\tau^\kappa$, with $0<\kappa<1$, and $\kappa\propto 1-g^{-1}$, where g is the conductance.

We have also considered the properties of a particle in an external potential, or moving around a closed ring. We find different qualitative behavior corresponding to the same two cases discused above:

i) If $\lim_{u\to\infty}\mathcal{F}'(u)$ decays faster than u^{-1}, a localized potential must exceed a threshold strength before localized solutions are possible. The energy scale which characterize the quantum properties of the particle is $\hbar^2/(M_{eff}R^2)$, for $R\gg l$, where l is a length scale which describes the range of the fluctuations in the environment. A model characterized by a function $\mathcal{F}$ which fluctuates between zero and a finite value as $u\to\infty$ can be included in this class[28].

ii) If $\lim_{u\to\infty}\mathcal{F}'(u)$ decays more slowly that u^{-1}, a localized solution exists for arbitrarily weak confining potentials. The characteristic energy scale which defines the quantum properties around a ring decays, for $R\gg l$, as $e^{-c(R/l)^{2\gamma}}$, where c and γ are constants.

In some cases, there is a first order phase transition,

as function of the strength of the potential, or the size of the orbit, between a weakly and a strongly localized solution.

Summarizing, we find that different ohmic environments can influence the quantum properties of an external particle in different ways: couplings with long range spatial interactions, such as in the Caldeira-Leggett model, strongly suppress quantum effects, while the effects of less singular couplings can be qualitatively understood within perturbation theory.

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